Technical Comments

Comment on "Some Approximations for the Dynamics of Spacecraft Tethers"

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THE paper by von Flotow[1] presents an excellent discussion of the effects of various factors such as curvature, stress-strain nonlinearity, and so forth, on the dynamics of tethers. It provides a useful insight that should offer guidance in setting up codes for numerical simulation in the future. This writer, however, disagrees with one conclusion reached in Ref. 1; specifically that the amplitude of transverse vibration remains more or less unchanged during deployment or retrieval of the subsatellite. von Flotow also mentions that his conclusion agrees with the simulation reported in Ref. 2. Actually, if one examines Fig. 7 of Ref. 2 one would notice that A_1, a generalized coordinate associated with out-of-plane transverse vibration, grows significantly during uncontrolled retrieval (from 95 to 200 m when the length changes from 100 to 4 km). The writer feels that it is quite important to recognize the existence of this growth, and it may be necessary, in many missions, to control this instability of transverse vibrations.

If one expands the transverse deflection as

$$u = \sum_{i=1}^{n} f_i(t)\phi_i(x,\ell) \qquad (1)$$

then the equation governing $\{f\}$ can be shown to be

$$\{\ddot{f}\} + 2(\dot{L}/L)[A]\{\dot{f}\} + [(\ddot{L}/L)[A] - (\dot{L}/L)^2[B] + (T/\mu L^2)[C]]\{f\} = \{F\} \qquad (2)$$

where the elements of $[A]$, $[B]$, and $[C]$ depend on some integrals of the shape functions ϕ_i. The second term is a damping-like term due to the Coriolis effects and the frequency-like quantity is basically $\sqrt{(T/\mu)}/L$. Thus, as von Flotow[1] rightly points out, the damping is small if $\dot{L}/L \ll \sqrt{(T/\mu)}/L$ or $\dot{L} \ll \sqrt{(T/\mu)}$. However, for the parameters used by von Flotow, $\sqrt{(T/\mu)} \simeq 90$ m/s while $\dot{L}$ may be as high as 5 m/s; thus, negative damping may be as high as 5% at some time. It is recognized, of course, that it is not quite correct to evaluate a damping coefficient this way as the coefficients are all time-varying and there is no such thing as a frequency or a damping coefficient for such systems. Arnold et al.[3] have shown that the transverse displacement changes as $L^{-1/4}$ by following a heuristic argument. This relation agrees quite well with numerical simulations (e.g., Fig. 7 of Ref. 2). Thus, u increases during retrieval, although slowly. A retrieval, say, from 2 km to 20 m implies a 5 1/2-fold increase in amplitude of oscillations. This may not appear great, but note that an initial amplitude of only 2 m becomes 11 m for the 20-m length. (Of course the linear model is no longer valid by then.)

Recently, the same $L^{-1/4}$ relation has been obtained by an approximate analysis of Eq. (2) in Ref. 4, although it appeared after von Flotow wrote his paper.

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References

[1]von Flotow, A. H. "Some Approximations for the Dynamics of Spacecraft Tethers," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 4, 1988, pp. 357–364.

[2]Misra, A. K. and Modi, V. J., "A Survey on the Dynamics and Control of Tethered Satellite Systems" *Advances in Astronautical Sciences*, Vol. 62, 1987, pp. 667–691.

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[4]Kalaycioglu, S. and Misra, A. K., "Analytical Expressions for Vibratory Displacements of Deploying Appendages," *Proceedings of the AIAA/AAS Astrodynamics Conference*, AIAA, Washington, DC, Aug. 1988, pp. 270–277.

Reply by Author to A. K. Misra

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PROFESSOR Misra disagrees with the estimate published in Ref. 1 that the amplitude of transverse vibration of a spacecraft tether remains approximately constant during slow retrieval. He bases his disagreement on simulation[2] and on two unpublished approximate analyses.[3,4] In preparing this response, I have obtained both these approximate analyses from the authors, and can report on the approximations made in Refs. 1, 3, and 4.

The estimate I published in Ref. 1 is based on the assumption (clearly stated) that the energy of the vibratory motion remains constant, and that the mode of vibration remains unchanged. Both Refs. 3 and 4 also assume that the mode shape remains unchanged; the source of the disagreement in the prediction is the assumption that the energy of vibration remains constant. If one is to believe the $L^{-1/4}$ amplitude-growth prediction, one must conclude that the energy of vibration increases during retrieval. It is certain that the retrieval machinery does work on the tether during retrieval; not so clear is the mechanism by which this work appears as energy of transverse vibration.

The estimate of Ref. 3 is based on heuristic arguments. The reader is asked to visualize a "skipping rope" motion of the tether about the line connecting the two attachment points. The mode shape of this deflection is assumed not to change as the tether shortens and whirls about this line. If one insists that the angular momentum of the tether about the line connecting the two attachment points is conserved, one is led to the conclusion that the amplitude of the lateral deflection grows with $L^{-1/4}$. For planar motion, where this angular momentum is zero, the analysis does not apply but the prediction is nevertheless assumed to be valid.

Reference 3 uses the same heuristic argument to estimate that the longitudinal vibrational motion of the tether shrinks during retrieval as $L^{1/4}$. The estimate of longitudinal motion published in Ref. 1, based on conservation of energy of longi-

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Table 1 Predicted amplitude ratios for comparison with Fig. 2 from Ref. 5

t(orbits)	$(L/L_o)^{-1}$	$(L/L_o)^{-5/4}$
0	1	1
0.8	2.54	3.2
1.6	5.63	8.7
2.4	13.3	25

tudinal vibration, is $L^{1/2}$. The estimates of Ref. 3 thus imply that the energy of lateral vibration grows during retrieval, whereas the energy of longitudinal vibration shrinks.

Reference 4 treats only lateral vibration by an approximate analysis of a governing linear partial differential equation. This is the piano string equation with constant tension, and with convective temporal derivative due to axial retrieval velocity, but with the assumption that the motion of the line connecting the two attachment points is fixed in a Newtonian reference frame; the terms involving rotation of the "tether frame" are ignored. Ignoring these terms was justified in Ref. 1 by the claim of spectral separation between the vibrational motion and the rotational motion of the system. Reference 1 further argues that approximating the convective derivative by ignoring the term due to retrieval rate is consistent with the assumption of spectral separation since the retrieval rate remains slow. It is this approximation that Kalaycioglu and Misra[4] do not make, and it leads them to the prediction that the amplitude of lateral vibrations grows as $L^{-1/4}$. Further mathematical approximations are made to derive this result, but they appear justified. The assumption that the lateral deformation consists of a single mode, with shape independent of length, is also made.

Finally, one can discuss the results of the published numerical simulations. Figure 7 from Ref. 2, used by Misra in his comment, indeed shows a slow growth of the two generalized coordinates A_1 and A_2 (presumably the amplitudes of two pinned-pinned sinusoidal shapes). This growth is consistent with the prediction that the amplitude grows according to $L^{-1/4}$; this prediction would lead to an increase in amplitude by a factor of 2.2 during a slow retrieval from 100 km to 4 km.

A comparison with the paper of Misra et al.[5] yields a different conclusion. This paper discretizes the lateral tether deflection with pinned-pinned sinusoidal shape functions, but scales the deflection with instantaneous length; out-of-plane deflection is given by

$$u(y,t) = L\sum_{i=1}^{n} A_i(t)\sqrt{2}\,\sin(i\Pi\, y/L) \tag{1}$$

No growth of lateral displacements would thus be indicated by a simulation in which A_i varies as L^{-1}. An $L^{-1/4}$ growth of lateral displacement would be indicated by A_i growing with $L^{-5/4}$. Unfortunately, Ref. 2 shows plots of $A_i(t)$, and the reader must attempt to recognize the difference between L^{-1} and $L^{-5/4}$ trends. I have attempted this, and would summarize this attempt with reference to Fig. 2 from Ref. 5 and Table 1 of this reply. The retrieval rate is relatively simple and slow:

$$\dot{L} = -2\times10^{-4}L \tag{2}$$

where T is the orbital period, taken to be 5400 s, so

$$L/L_0 = \exp\,(-1\cdot08\ t/T) \tag{3}$$

It is clear from inspection of Fig. 2 that neither A_1 nor A_2 grows even as quickly as L^{-1}, rather much more slowly. This implies that the lateral displacements, proportional to $L\ A_1$ and $L\ A_2$, shrink.

Conclusions

References 3 and 4 predict that lateral tether vibration amplitudes grow with $L^{-1/4}$ during retrieval. This prediction appears well-matched by a simulation result imprecisely reported in Ref. 2.

Reference 1 predicts that lateral tether vibration amplitudes remain approximately constant during slow retrieval, if energy is conserved.

Reference 5 reports a numerical simulation which predicts that lateral tether vibration amplitudes shrink during slow retrieval, but that this amplitude shrinks slightly more slowly than the length.

It is apparent that there is some disagreement.

References

[1]von Flotow, A. H., "Some Approximations for the Dynamics of Spacecraft Tethers," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 4, 1988, pp. 357–364.

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[3]Arnold, D. A., "Tether Tutorial," International Conference on Tethers in Space, Venice, Sept. 1988.

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Comment on "Efficacy of the Gibbs-Appell Method for Generating Equations of Motion for Complex Systems"

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Introduction

REFERENCE 1 is the latest in a series of papers by Professor Desloge (see Refs. 2–4) on the Gibbs-Appell method for formulating equations of motion and its relationship to Kane's method. Like the previous papers, the present one contains invalid statements. Although Desloge's claims concerning the independence and relative merits of these two methods have been refuted previously (see, for example, Refs. 5–10), the defects of his latest paper are so serious that it is necessary to set the record straight once more.

In Ref. 1, Desloge applies the Gibbs-Appell method to the formulation of equations of motion of a system consisting of a rigid body and a particle whose motion relative to the body is subject to arbitrary kinematical constraints. He then specializes these equations to those given by Kane and Levinson[11] for a rigid body carrying a light four-bar linkage on whose coupler bar a particle is free to slide. Desloge thereupon claims, without substantiation, that it requires less labor to obtain the equations of motion in this manner than by employing Kane's method as in Ref. 11. This claim has no substance because the

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